Supplementary Materials of “Feasible Region Characterization of Temporal Coupling Technical VPP: A Reversible Decoupling Projection Approach”

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A. Proof of Absorptivity and Distributivity of the Convex Set Projection

1. ***Proof of ***

We demonstrate the absorptivity of the convex set projection, that is the addition of projection variables  does not affect the FR of the original projection variables , meaning that the projection of the  on  remains identical to. The mathematical expression can be formulated as: . We proceed with a proof by contradiction. Assume that the aforementioned equality does not hold. Then, there exists  such that either  is satisfied, or exists  such that either  is satisfied. Since the proof strategies for both cases are analogous, we only present the proof for the former scenario. By the definition of the projection, due to , there must exist a *y* such that . Moreover, since  contains all feasible points of *y* and *z* in ,  necessarily follows. Similarly, as  encompasses all feasible points of *z* in ,  is also satisfied. However, this conclusion contradicts the original proposition. Therefore, our initial assumption must be false, and the original proposition holds.

1. ***Proof of ***

Since , . The following is to prove the intersection distributivity of convex set projection, that is if  and  are closed convex sets and , . It is worth noting that, since  contains only the variable *y*, its cardinality is infinite. However, the  in this letter possesses certain special characteristics. Specifically, a cuboid closed region, with dimensions far exceeding the limit values of *y* and *z*, can be artificially defined as the feasible boundary for *y* and *z*. This ensures that  retains its closed convex set property. It should be emphasized that imposing this constraint does not affect the actual feasible region of *y* and *z* after the intersection operation. The proof of  requires  and . On the one hand, if , where , then there exists  such that , and  and . Therefore,  and , that is, . On the other hand, since  and  are closed convex sets and their intersection is non-empty, for any , there exist  and  such that  and . Furthermore, since  and  are closed convex sets, their vertical sections  and  are closed convex sets. Since both  and  are non-empty closed convex sets and , then there exists a common point . Therefore, there exists a  such that , that is, .

1. ***Proof of ***

The proof of  is equivalent to proving the validity of the union distributivity of convex set projection, that is . Similarly, the bidirectional inclusion relationship needs to be verified:  and . On the one hand, let . Then there exists a point . According to the definition of the union,  or . If , then . If , then . Therefore, . On the other hand, let . Then  or . If , then there exists a point . Hence, . If , then there exists a point . Hence, .

B. The Parameter Configurations of the Case Study

The accuracy of the proposed FRC method for TVPP is verified on an improved IEEE 33 node test feeder. To validate the adaptability of the proposed method in handling varying numbers of DERs with temporal coupling constraints (ES and GT), we designed two distinct sets of cases:

**case1**: case1 comprises 3 PVs, 6 WTs, 10 GTs, 1 SVCs and 1 ESs. The PVs are connected to nodes 4, 5, and 7; the WTs are integrated at nodes 11, 15, 19, 24, 28, and 32; the GTs and SVCs are connected to nodes 3, 5, 9, 12, 14, 16, 19, 21, 26, and 29; and the ESs are deployed at nodes 5, 7, 11, 14, 19, 21, 26, 28, 30, 31, and 33.

**case2**: case2 comprises 3 PVs, 6 WTs, 10 GT, 10 SVCs and 10 ES. The PVs are connected to nodes 4, 5, and 7; the WTs are integrated at nodes 11, 15, 19, 24, 28, and 32; the GT is connected to node 14; the SVCs are connected to nodes 3, 5, 9, 12, 14, 16, 19, 21, 26, and 29; and the ES is deployed at node 26.

The parameters for the GTs and ESs are referenced from [1] and the installed capacity of DERs and the load of TVPP are shown in Table I. The daily source-load forecast data is derived from publicly available datasets in the EU and neighboring regions [2]. The confidence level  is set to 0.9 which has been used in [3]. The upper and lower voltage limits are set to 1.05 p.u. and 0.95 p.u. The improved IEEE 33 node test feeder which is illustrated in Fig. 1. In addition, case studies are performed using MATLAB R2023a and GUROBI on a PC platform equipped with 16GB of RAM and an Intel Core i7-12700H CPU (2.70 GHz).



Fig. 1 IEEE 33 topology.

TABLE I

The installed capacity of DERs and the load of TVPP

|  |  |  |
| --- | --- | --- |
| Parameters | | Value |
| Maximum Active Load of TVPP (MW) | | 3.54 |
| Maximum Reactive Load of TVPP (MVar) | | 2.19 |
| Installed Capacity of case1 | PV&WT (MW) | 0.50-2.00 |
| GT (MW) | 1.80 |
| SVC (MVar) | 0.15-0.30 |
| ES (MWh) | 2.20 |
| Installed Capacity of case2 | PV&WT (MW) | 0.50-2.00 |
| GT (MW) | 0.18-0.28 |
| SVC (MVar) | 0.15-0.30 |
| ES (MWh) | 0.36-0.80 |

Reference

[1] A. Jani and S. Jadid, "Two-stage energy scheduling framework for multi-microgrid system in market environment," *Applied Energy*, vol. 336, Apr 2023, Art no. 120683, doi: 10.1016/j.apenergy.2023.120683.

[2] Open Power System Data. 2020. Data Package Time series. Version 2020-10-06. [Online]. Available:[https://doi.org/10.25832/time series/2020-10-06](https://doi.org/10.25832/time%20series/2020-10-06)

[3] Y. Cao, W. Wei, S. Mei, M. Shafie-khah, and J. P. S. Catalão, “Analyzing and Quantifying the Intrinsic Distributional Robustness of CVaR Reformulation for Chance-Constrained Stochastic Programs," *IEEE Trans. Power Syst*., vol. 35, no. 6, pp. 4908-4911, 2020.